INELASTIC BEHAVIOUR OF FRAME STRUCTURES UNDER STATIC AND EARTHQUAKE FORCES

by

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SYNOPSIS

The effects of a broad range of variables on the response of inelastic multi-storey structures subjected to quasi-static and earthquake forces are studied. The characteristics of static collapse are identified and discussed with reference to the implications for earthquake performance. The significance of yielding during earthquake excitation is examined by comparison to corresponding elastic behaviour. Attention is directed toward the energy demand, magnitude and distribution of maximum response, and possible overall collapse. Finally, to relate in a simple manner the effective intensity of a given earthquake to the expected magnitude of inelastic response, various definitions of intensity are considered.

GLOSSARY

C_y	= static base shear coefficient at first yield
EI	= flexural rigidity
E_D	= energy dissipated by viscous damping per unit weight of structure
E_S	= stored elastic strain energy per unit weight of structure
E_T	= earthquake energy input per unit weight of structure
E_Y	= energy dissipated by yielding (hysteretic energy) per unit weight of structure
f_n	= normalized horizontal force at level 'n'
g	= gravitational constant
m	= uniform lumped storey mass
M_y	= member yield moment = plastic moment capacity

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$\bar{s}$ = bi-linearity coefficient of moment-rotation relationship

SI_{ζ} = spectrum intensity

$S_v^{(j)}(\zeta_j)$ = maximum pseudo-velocity associated with j^{th} mode

T = fundamental period of vibration

$\ddot{y}(t)$ = ground acceleration

$\bar{\ddot{y}}(T_D)$ = root-mean-square ground acceleration for duration T_D

γ = ratio of root-mean-square seismic force to lateral yield strength of structure

Δ = incremental value when followed by variable name; otherwise, 'total sway' defined as floor displacement divided by height of structure

δ = 'inter-storey sway' defined as relative displacement of adjacent floors divided by storey height

ζ_j^a = absolute viscous damping ratio of j^{th} mode

ζ_j^r = relative viscous damping ratio of j^{th} mode

μ = ductility factor

ρ_M = factor for transverse girder loading

ρ_P = factor for 'P-Δ' forces

INTRODUCTION

Analytical investigations, as well as the performance of structures during past earthquakes, have demonstrated the importance of inelastic behaviour for the survival of structures subjected to strong earthquake motions. Reflecting this experience, it has been suggested [1,2] that aseismic design consist of two independent phases: (a) elastic design for moderate ground motions, and (b) plastic design for strong ground motions. Design according to condition (a) is readily accomplished using well-established principles in the theory of structural dynamics. Limit design procedures, for use with condition (b), have also been proposed based on energy considerations [1,2,3]. The need for simplicity in practical design requires that these take the form of pseudo-static methods. To permit such an approach to the design of multi-storey structures, detailed knowledge of the characteristics of inelastic behaviour, for both static lateral loading as well as strong earthquake excitation, must be developed.

The work presented in this paper is part of a broader investigation [4] into the inelastic behaviour of frame structures under static and earthquake forces. To assure a unified and comprehensive approach, three complementary problems were considered: (1) development of a class of model structures; (2) mechanics of collapse under static lateral loading; and (3) behaviour under earthquake excitation. Major emphasis in this paper is given to results from the dynamic portion of the investigation.

STRUCTURAL MODELS

The class of structures considered in this study is characterized by the rigid moment-resisting frame. A system of model frames, simply described by five idealized parameters, was developed. Frame geometry is constant: bay size, L = 20 ft; uniform storey height, h = 12 ft; number of storeys, N = 10; and equal lumped storey masses, m. A general model of this class is shown in Fig. 1.

The structural properties of members are linear functions of height. Girder and column flexural rigidities at level 'n' may, respectively, be written as

$$
\begin{aligned}
EI_{G_n} &= \xi_{G_n} EI_o \\
EI_{C_n} &= C_C \, \xi_{C_n} EI_o
\end{aligned}
\qquad (1)
$$

where ξ_{G_n} and ξ_{C_n} are the normalized distribution functions,

$$
\begin{aligned}
\xi_{G_n} &= 1 + \frac{(n - 1)(\tau_G - 1)}{(N - 1)} \\
\xi_{C_n} &= 1 + \frac{(n - 1)(\tau_C - 1)}{(N - 1)}
\end{aligned}
\qquad (2)
$$

and the quantity, EI_o, is the reference flexural rigidity. Similar relationships relate member strength, M_y, to the reference yield strength, M_{yo}. The relation between flexural rigidity and moment capacity selected corresponds to 14 WF sections.

Analysis of an individual member of this system is based on the assumption of 'unit shape factor'. This means that hinges form at critical sections defined here as occurring at each end of the member and consisting of a rigid-plastic Coulomb element in parallel with a linear spring. Overall member behaviour, expressed in terms of end moment-end rotation relationships, is described by the bi-linear model shown in Fig. 2. The application of this model to hysteretic member behaviour has been discussed [4,5,6].

To evaluate model parameters appropriate to a particular prototype structure, a 10-storey single-bay frame was chosen and designed according to the National Building Code of Canada for Zone 3 seismic exposure. Member selection was based on the use of ASTM A36 steel, with design gravity loading of 55 kips dead load and 37.5 kips live load per floor, uniformly distributed. Values for the parameters of the model system related to this typical design are given in Table I. All quantitative data presented herein were obtained with these values for the system parameters. Detailed discussions of behaviour for other models, with parameters varied over a range of values applicable to structures encountered in practice, are presented in Ref. [4].

VISCOUS DAMPING

Two models of the viscous damping mechanism are considered. The first consists of dashpots connecting each floor mass to the ground. This model, termed the 'absolute' model, is assumed in the form proportional to mass, with the associated damping matrix given by

$$[C^a] = \alpha[M] \tag{3}$$

where α is a scalar constant and [M] is the diagonal mass matrix. The j^{th} modal damping ratio obtained with this mechanism, for frequency ω_j, decreases with the higher modes and is given by

$$\zeta_j^a = \frac{\alpha}{2\omega_j} \tag{4}$$

The second model of the viscous damping mechanism consists of dashpots connecting adjacent floors. This mechanism is the 'relative' model, and the elements of the associated tri-diagonal damping matrix, $[C^r]$, are easily formed from the dashpot constants. For a particular modal damping ratio, ζ_j^r, the dashpot constants can be established on the basis of harmonic motion and a per cycle energy dissipation equal to that of the absolute model. Instead of determining a complete solution in a 'least squares' sense, an approximate solution using the properties of the fundamental mode only was adopted. It is easily shown that the resulting mechanism operates approximately as the stiffness proportional model of the 'Rayleigh' type [7] for which

$$\zeta_1^r < \zeta_2^r < \ldots < \zeta_N^r$$

STATIC ANALYSIS

Load Factors: The loading condition used for the study of static collapse consists of horizontal forces of varying magnitude together with prescribed constant vertical forces. This non-proportional loading system is defined in terms of three independent load factors:

1. ρ_M = factor for constant girder transverse loading;
2. ρ_P = factor for constant vertical gravity forces;
3. p = factor for variable horizontal forces.

Factors ρ_M and ρ_P are applied to the design dead load. In terms of response phenomena, ρ_M represents the magnitude of primary moments and ρ_P indicates the level of the forces involved in the so-called 'P-Δ' effect. When only certain bents of a frame building comprise the lateral load resisting system, transverse girder loads form only part of the 'P-Δ' forces. Consequently, the factors for these loads are treated as independent parameters. The parameter, p, denotes the variable magnitude of the horizontal forces,

$$\{F\} = p\{f\} \tag{5}$$

where {f} is the normalized vector describing the distribution pattern. A similar loading condition was employed by Tanabashi et al. [8].

Method of Analysis: The static collapse behaviour of a given structure is examined in terms of its equilibrium path in the load-displacement plane. This equilibrium path is obtained from successive solutions of the incremental equilibrium equation,

$$([K]) - [G])\{\Delta q\} = \{\Delta Q\} \tag{6}$$

where [K] is the incremental structural stiffness matrix, [G] is the instability matrix representing the 'P-Δ' forces, and $\{\Delta q\}$ and $\{\Delta Q\}$ are vectors of incremental displacements and forces, respectively. The influence of axial force on member stiffness is neglected and, since analysis based on a prescribed load increment will encounter certain difficulties beyond the failure load, the analytical procedure was formulated in terms of a prescribed top floor displacement.

DYNAMIC ANALYSIS

Equations of Motion: The differential equations of motion, in incremental matrix form, are given by

$$[M]\{\Delta\ddot{q}\} + [C]\{\Delta\dot{q}\} + ([K] - [G])\{\Delta q\} = \{\Delta Q\} \tag{7}$$

where 'dots' denote differentiation with time and [C] is the viscous damping matrix. Since the 'q' co-ordinate system is defined with respect to the base of the structure, these equations represent motion relative to the ground; thus $\{\Delta Q\}$ becomes the vector of incremental seismic forces. Equations 7 were solved using a direct numerical integration procedure, assuming a linear variation of acceleration over small intervals of time. The complete solution for the motion of the system is obtained by superimposing the successive incremental solutions. This procedure was developed originally by Clough and Wilson [9].

Energy Relationships: Conservation of energy yields the rate equation,

$$\dot{E}_T(t) = \dot{E}_S(t) + \dot{E}_D(t) + \dot{E}_Y(t) + \dot{E}_K(t) \tag{8}$$

where E_T, E_S, E_D, E_Y and E_K represent, respectively, energy input by seismic forces, recoverable elastic strain energy, energy dissipated by viscous damping, energy dissipated by yielding and relative kinetic energy. For convenience, these quantities are expressed per unit total weight of the structure and termed 'unit energies'. The terms related to gravity forces are not shown to restrict attention to the earthquake energy input. Integration of Eq. 8 was accomplished by a step-by-step procedure, wherein the incremental energy quantities are evaluated according to the linear variation of acceleration assumed in solving Eq. 7.

For elastic systems possessing normal modes, an approximation for the maximum strain energy may be obtained by assuming the maximum pseudo-velocity for each mode, $S_v^{(j)}(\zeta_j)$, equal to that of the fundamental mode. Hence,

$$\max E_S(t) \simeq \frac{1}{2g} [S_v^{(1)}(\zeta_1)]^2 \tag{9}$$

This estimate for the maximum elastic strain energy also provides an approximation for the maximum internal energy, $\max(E_T - E_D)$. For inelastic behaviour, the energy demand is expressed in terms of the maximum energy absorbed in the structural system, $\max(E_y + E_S)$.

Definitions of Intensity: Three independent measures for the intensity of ground acceleration, $\ddot{y}(t)$, are frequently employed:

1. Peak ground acceleration, $\max|\ddot{y}|$;

2. Root-mean-square ground acceleration, $\bar{\ddot{y}}$;

3. Spectrum intensity, SI_ζ.

Here, ζ denotes the viscous damping ratio associated with the pseudo-velocity spectrum. These measures of intensity are independent of the properties of the structure under consideration and, consequently, cannot be related directly to the magnitude of inelastic response.

For use with yielding systems, any meaningful measure of intensity must relate the level of excitation to the strength of the structure. Therefore, the following relative intensity factor is introduced:

$$\gamma = \frac{\bar{\ddot{y}}}{g\, C_y} \tag{10}$$

where C_y is the coefficient of base shear at first yield for uniform horizontal loading.

Since frequency is an important parameter in dynamic response, the spectral pseudo-velocity of the fundamental mode, $S_v^{(1)}(\zeta_1)$, as well as the corresponding maximum energy, $[S_v^{(1)}(\zeta_1)]^2/2g$, may be expected to provide useful measures of intensity. The success with which these various measures of intensity predict the magnitude of inelastic response for different earthquake records is examined.

RESULTS PRESENTED

Definition of Terms: The maximum deformation of a member is discussed in terms of the ductility factor, μ, given by

$$\begin{aligned} \mu &= \max\left|\frac{\theta}{\theta - \alpha}\right|, \text{ where yielding has occurred} \\ &= \max\left|\frac{M}{M_y}\right|, \text{ otherwise} \end{aligned} \tag{11}$$

where θ, α, and M denote, respectively, total end-rotation, yielded hinge rotation and total end-moment.

The results for quasi-static loading are presented in the form of non-dimensional load-displacement diagrams, as well as envelopes of maximum ductility factors plotted over the height of the structure. The latter apply to the displacement associated with the peak in the load-displacement diagram.

The results for inelastic response to earthquake excitation are examined in the form of envelopes of maximum absolute response plotted over the height of the structure. The data related to displacements are presented in terms of the total sway, Δ_n, indicating the ratio of maximum absolute displacement at level 'n' to the overall height of the structure, as well as the inter-storey sway, δ_n, denoting the ratio of maximum absolute displacement between adjacent floors to the corresponding storey height.

Standard Data: In an attempt to isolate the effect of a particular parameter, the procedure employed consists of varying a single parameter while maintaining all others constant at the standard values given in Table II. The standard form of excitation consists of the first 15 seconds of the El Centro earthquake of May 18, 1940, N-S component magnified by a factor of 1.5.

DISCUSSION OF RESULTS

Static Collapse Behaviour: Figure 3 shows the effect of the horizontal loading pattern, {f}, on the results from quasi-static analyses. These results indicate that the general nature of static collapse behaviour is not sensitive to widely differing loading patterns. Uniform horizontal forces are, therefore, a reasonable approximation in studying the characteristics of static behaviour.

The variation in static behaviour for differing intensities of primary moments, ρ_M, is presented in Fig. 4. Primary moments have only a small influence on the sidesway buckling loads of elastic frames, and the curves of Fig. 4(a) indicate a similar conclusion for horizontal loading and elastic-plastic structures. Here, the change in ρ_M from 0 to 3.0 has reduced the failure load from 0.172 to 0.163, a reduction of only 5.2 per cent.

The most significant effect of primary moments involves the ductility distributions of Fig. 4(b). Premature yielding, the result of primary moments, leads to increased ductility factors at static collapse, particularly in the upper portion of the structure. The maximum ductility factor increases from 2.9 to 6.2 at the third floor level, whereas at the top floor the corresponding change is from 0.2 to 2.9, for variation in ρ_M from 0 to 3.0

The influence of the 'P-Δ' forces on static collapse is given in Figs. 5 and 6. It is noted that ρ_P tends to confine inelastic action to progressively lower portions of the structure [Figs. 5(b) and 6(a)] while, at the same time, appreciably reducing the horizontal load capacity [Fig. 5(a)]. The greatest changes, however, occur in the maximum ductility factors of Fig. 5(b), where an increase in ρ_P from 0 to 3.0 decreases the maximum ductility factor from 12.1 to 1.5. It is apparent that the simple first-order analysis, given by $\rho_P = 0$, yields little useful information regarding the actual characteristics of collapse. A notable exception concerns the manner in which inelastic action occurs; the order of hinge formation (Fig. 6) is unaltered by gravity forces, although the total number of hinges formed decreases with increasing ρ_P.

On the basis of the static behaviour discussed above, and additional results given in Ref. [4], the following predictions for dynamic response are noted. Vertical forces may be expected to increase inelastic dynamic response because of reduced lateral strength and higher degree of instability in the post-buckling region (as indicated by the negative slopes of the load-displacement curves). Primary moments, on the other hand, cause early yielding of members, particularly upper level girders, and the increased hysteretic energy dissipation of this region may result in a decreased energy demand for the lower portion of the structure, thereby suggesting a possible reduction in overall response.

Static Forces and Earthquake Response: The diagrams of Fig. 7 illustrate the variation in dynamic response for increasing 'P-Δ' forces. The presence of gravity forces can affect inelastic dynamic response appreciably, depending on the magnitude of ρ_P. Forces due to the design dead load ($\rho_P = 1.0$) have produced maximum increases in the displacement responses of approximately 15 per cent [Figs. 7(a) and 7(b)]. The distribution of hinging action associated with these diagrams is shown in Fig. 6(b). Comparison of the hinge formation patterns for $\rho_P = 0$ and $\rho_P = 1.0$ indicates that this feature of response is not altered by the dead load gravity forces. However, further increases in gravity loading result in formation of a six-storey sway mechanism in the lower portion of the structure. The curves of Figs. 7(a) - 7(d), reflecting the action of this mechanism, indicate a corresponding increase in response for this portion of the structure. It is of interest to note that the dynamic mechanism is the same as the first-order static collapse mechanism (Fig. 6).

Idealization of the frame structure as a six-storey mechanism consisting of rigid bars and elastic-plastic springs allows evaluation of the danger of overall collapse. It is found that, in the absence of lateral supporting forces, this sytem will collapse at the non-dimensional top displacement, $\Delta_S = 0.120/\rho_P$. Thus, the computed response ratio, max $\Delta/\Delta_S = 0.5$ for $\rho_P = 3.0$, indicates that dynamic overall collapse is not imminent. On the basis of this, as well as other data given in Ref. [4], it is concluded that dead load gravity forces require an unrealistic intensity and/or duration of excitation to cause overall collapse for structures of the class considered here.

Figure 8 summarizes the characteristics of dynamic response for increasing magnitude of primary moments. Figures 8(a) and 8(b) confirm that the increased hysteretic energy dissipation due to the early yielding of upper level girders leads to the reduction in response predicted by the static analysis. For primary moments associated with dead load, the top displacement and the maximum storey sway are reduced by 2.5 and 8.0 per cent, respectively. The maximum ductility factors, on the other hand, increase with increasing ρ_M. A comparison of Figs. 8(c) and 4(b) verifies that the static analysis predicts the effect of ρ_M on the dynamic ductility distributions reasonably well, with the exception that dynamic excitation causes accentuated yielding in the upper portion of the structure. Thus, the influence of dead load primary moments on the overall dynamic response is small. Omission of these forces may be expected to provide conservative (larger) estimates for maximum response generally, but individual member ductility factors may be under-estimated to a considerable extent.

Intensity of Excitation and Inelastic Response: Figure 9 shows the envelopes of maximum dynamic response for levels of intensity from the limit of elastic response to a level in excess of the most intense ground motion expected to occur. The value, $\gamma = 0.26$, represents approximately 50 per cent El Centro intensity, whereas $\gamma = 0.78$ corresponds to Housner's estimate [10] for the upper bound on expected ground motion. The elastic limit, when max $\mu = 1.0$, occurs for $\gamma = 0.26$ as noted above; hence, this value is termed the yield intensity, γ_y.

The trend generally noted is that inelastic action is confined principally to the girders of the structure. The columns remain elastic for $\gamma \leq 0.52$; however, for more intense excitation, extensive yielding of the girders causes the columns to act effectively as free standing cantilevers, which in turn leads to yielding of the column bases [Fig. 9(b)]. Figure 9(a) indicates that formation of these column base hinges, for $\gamma \geq 0.78$, is accompanied by a pronounced increase in displacements. Detailed examination of these diagrams reveals that increasing the intensity of excitation creates a tendency for inelastic action in the lower portion of the structure to suppress the contributions of the higher modes, causing the response envelopes to resemble in progressively greater degree the shapes characteristic of the elastic fundamental mode.

The relative significance of the hysteresis and the viscous damping mechanisms for increasing intensity of excitation is given in Fig. 10. It is noted that hysteretic behaviour becomes the principal dissipative mechanism when $\gamma/\gamma_y \approx 1.5$, for the degree of viscous damping, $\zeta_1^a = 0.015$, employed here. The effect of duration of excitation beyond the 15 seconds used in the present analyses consists of an upward displacement given by the broken line. The latter represents an estimate obtained from a linear rate of viscous energy consumption with time. These energy curves indicate that yielding increases in importance for excitation of high intensity and short duration.

Table III compares the variation in maximum inelastic response, for three different accelerograms, with the estimates of effective intensity according to the various definitions discussed previously. The data indicate that the measure of intensity given by the energy-based criterion, $[S_v^{(1)}(\zeta_1)]^2/2g$, provides the most accurate prediction for the magnitude of inelastic response.

<u>Viscous Damping and Vibrational Energy Dissipation</u>: In Fig. 11 two solutions of the energy equation for different viscous damping ratios are shown as functions of time. The variation of energy input with time, $E_T(t)$, as well as its maximum value, appears to be largely independent of the mechanics of internal energy consumption. The hysteretic energy dissipation is characterized by a small number of discrete lurches, followed by plateaus of elastic behaviour. The energy dissipated by viscous damping, on the other hand, increases almost linearly with time. The fraction of the maximum energy input that is dissipated by either of these mechanisms is given in Fig. 12 as a function of the damping ratio, ζ_1^a. Regardless of the extent to which either mechanism operates, 92 - 99 per cent of the maximum energy input is dissipated by the structure during the first 15 seconds of the excitation.

Figure 13 compares the capacity for energy dissipation of the absolute and relative viscous damping models, with only one of these models assumed for the structure. The relative model is seen to absorb a significantly greater proportion of the total energy dissipated, suggesting that relative damping is accompanied by reduced response. The associated maximum response diagrams [4] confirm that the relative damping model yields lower values of storey sways, absolute accelerations, storey shears, ductility factors and related quantities. Total floor displacements, however, are only mildly affected by the type of viscous mechanism.

These observations are due mainly to the differences in the damping ratios of higher modes resulting from the two models of viscous damping. Significantly, experimental determination of damping coefficients [7] seems to suggest that a combination of the relative and absolute models is required.

Although accurate determination of response involves knowledge of the appropriate viscous mechanism, for purposes of design the absolute model may be employed to provide conservative estimates of response.

Effect of Yielding: The response data of Table IV demonstrate the effect of yielding for increasing intensity of excitation. The maximum internal energy, $\max(E_T - E_D)$, is approximately the same for inelastic and elastic behaviour. Similarly, the spectral estimate for maximum elastic strain energy, $[S_v^{(1)}(\zeta_1)]^2/2g$, is found to overestimate slightly the maximum inelastic energy demand, $\max(E_y + E_S)$. A somewhat greater reduction in maximum displacement results due to yielding. These trends appear to be virtually independent of the intensity of excitation.

Table V tabulates the effect of yielding as a function of the natural period of the elastic fundamental mode. The data indicate that the effect of yielding is strongly dependent on the frequency of the system but, since the variation is not systematic, no particular significance can be attached to the individual cases considered. These observations are in agreement with similar results [3] obtained for single-degree-of-freedom systems.

Table VI presents the effect of yielding for different degrees of viscous damping. It is seen that the ratios of maximum inelastic to maximum elastic response, for both energy and displacement, increase with the damping ratio, ζ_1^a. For the case with no viscous damping, yielding results in a 30 - 35 per cent reduction in response. With small amounts of viscous damping, inelastic and elastic response are approximately equal; however, for viscous damping of 10 per cent, inelastic behaviour is accompanied by an appreciable increase in maximum response.

Approximations for Inelastic Response: To arrive at results that are meaningful for use with general, rather than particular, cases the various examples presented here are treated as samples in a Monte Carlo approach. Thus, for different intensities, fundamental frequencies and amounts of viscous damping, approximations for inelastic behaviour are obtained:

1. An estimate for the maximum energy stored in the inelastic structural system is given by

$$\max(E_y + E_S) = \frac{1}{2g}[S_v^{(1)}(\zeta_1)]^2$$

with a maximum deviation of +56 per cent and a mean deviation of -5 per cent.

2. An estimate for the maximum inelastic displacements is given by the maximum displacements associated with elastic behaviour, with a maximum deviation of +32 per cent and a mean deviation of -10 per cent.

SUMMARY AND CONCLUSIONS

Representative data illustrating the character of the inelastic behaviour of a model for a typical frame structure subjected to both static and earthquake forces have been presented in this paper. The following is a brief summary of the major trends observed and, it is believed, additional studies would indicate a broader interpretation of these results.

1. The presence of primary moments due to transverse girder loading has only a small effect on the dynamic response generally; however, individual member ductility factors may be increased considerably, particularly in the upper levels of the structure.

2. The gravity forces comprising the 'P-Δ' effect can have a significant influence on the inelastic dynamic response. Large vertical forces may cause the formation of a potential collapse mechanism, but overall collapse for the class of structures considered in this study does not appear probable for a realistic level and duration of excitation.

3. A comparison of the results obtained from parallel static and dynamic analyses indicates that knowledge regarding the collapse behaviour under quasi-static horizontal forces provides a useful basis for predicting characteristics of dynamic response.

4. The effect of viscous damping on inelastic response decreases with increasing intensity, and decreasing duration, of excitation. Although the maximum displacements are approximately equal for relative and absolute damping, the absolute model may be employed to provide conservative estimates for most response parameters of interest.

5. Over the range of parameters investigated, the results of this study indicate the tentative conclusion that approximations for the maximum displacement and the maximum energy demand associated with inelastic behaviour may be obtained from the damped elastic response spectra.

6. Of the various measures examined for the effective intensity of excitation, the elastic spectral energy associated with the fundamental mode provides the most satisfactory basis for relating the intensity of a particular earthquake to the magnitude of inelastic response.

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TABLE I PROPERTIES OF 10 - STOREY MODEL FRAME (FIG. 1).

EI_o ($lb - in^2$)	M_{yo} (lb - in)	m ($lb\text{-}sec^2/in$)	T (sec)	τ_G	τ_C	c_c	$\bar{s}$
$1{\cdot}88 \times 10^7$	$3{\cdot}44 \times 10^3$	0·143	2·1	1·5	3·0	0·7	1×10^{-5}

TABLE II STANDARD DATA FOR PARAMETRIC STUDY.

STATIC FORCES			PROPERTIES OF STRUCTURE			EARTHQUAKE FORCES	
ρ_M	ρ_P	f_n	c_y	ζ_1^a	ζ_1^r	$\bar{\ddot{y}}/g$	γ
0	$1{\cdot}0^{\dagger}$, $0^{\dagger\dagger}$	$1{\cdot}0^{\dagger}$, $0^{\dagger\dagger}$	$0{\cdot}157^{*}$	0·015	0	0·082	0·783

† STATIC COLLAPSE STUDY. †† EARTHQUAKE RESPONSE STUDY.

* WITH $\rho_M = 0$, $\rho_P = 0$ AND $f_n = 1{\cdot}0$.

TABLE III VARIOUS DEFINITIONS OF INTENSITY AND MAGNITUDES OF INELASTIC RESPONSE.

EARTHQUAKE RECORD	MEASURE OF INTENSITY					MAGNITUDE OF RESPONSE		
	$\max\lvert\ddot{y}\rvert$	$\bar{\ddot{y}}$	SI_o	$S_v^{(1)}(\zeta_1)$	$\frac{1}{2g}[S_v^{(1)}(\zeta_1)]^2$	max Δ	max δ	max μ
EL CENTRO 1940, N-S	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000
TAFT 1952, S21°W	1·047	1·000	0·825	0·640	0·410	0·419	0·515	0·487
JENNINGS ARTIFICIAL #1	0·810	1·000	0·913	0·711	0·505	0·503	0·546	0·517

NOTE: QUANTITIES HAVE BEEN NORMALIZED WITH RESPECT TO CORRESPONDING VALUES FOR THE EL CENTRO CASE.

TABLE IV INTENSITY OF EXCITATION AND THE EFFECT OF YIELDING.

INELASTIC RESPONSE / ELASTIC RESPONSE	RELATIVE INTENSITY			
	$\gamma/\gamma_y = 1{\cdot}00$	$\gamma/\gamma_y = 2{\cdot}04$	$\gamma/\gamma_y = 3{\cdot}06$	$\gamma/\gamma_y = 4{\cdot}08$
$\frac{\max(E_T - E_D)}{\max(E_T - E_D)^e}$	1·00	1·04	1·02	0·98
$\frac{\max(E_y + E_S)}{\frac{1}{2g}[S_v^{(1)}(\zeta_1)]^2}$	0·88	0·90	0·91	0·87
$\frac{\max \Delta}{\max \Delta^e}$	1·00	0·71	0·83	0·71

TABLE V FUNDAMENTAL PERIOD AND THE EFFECT OF YIELDING.

INELASTIC RESPONSE / ELASTIC RESPONSE	FUNDAMENTAL PERIOD, secs.			
	T = 2·1	T = 1·5	T = 1·0	T = 0·5
$\frac{\max(E_T - E_D)}{\max(E_T - E_D)^e}$	1·02	1·60	0·63	1·91
$\frac{\max(E_y + E_s)}{\frac{1}{2g}[S_v^{(1)}(\zeta_1)]^2}$	0·91	1·56	0·59	0·84
$\frac{\max\Delta}{\max\Delta^e}$	0·82	1·32	0·68	0·69

TABLE VI VISCOUS DAMPING RATIO AND THE EFFECT OF YIELDING.

INELASTIC RESPONSE / ELASTIC RESPONSE	DAMPING RATIO			
	$\zeta_1^a = 0$	$\zeta_1^a = 0{\cdot}015$	$\zeta_1^a = 0{\cdot}05$	$\zeta_1^a = 0{\cdot}10$
$\frac{\max(E_T - E_D)}{\max(E_T - E_D)^e}$	0·68	1·02	1·41	1·31
$\frac{\max(E_y + E_s)}{\frac{1}{2g}[S_v^{(1)}(\zeta_1^a)]^2}$	0·61	0·91	1·06	1·32
$\frac{\max\Delta}{\max\Delta^e}$	0·65	0·82	1·25	1·24

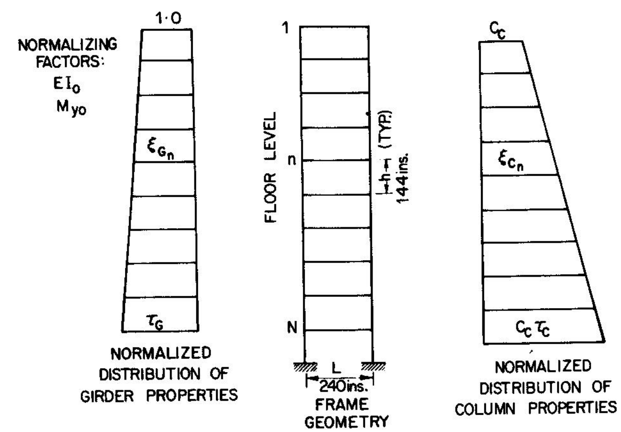

FIG. I PROPERTIES OF MODEL STRUCTURES

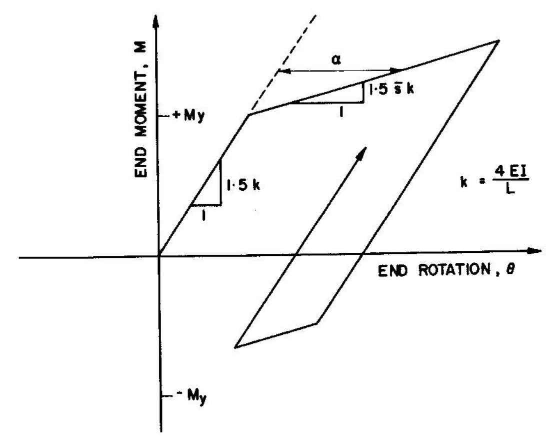

FIG. 2 BI-LINEAR HYSTERETIC MOMENT-ROTATION RELATION

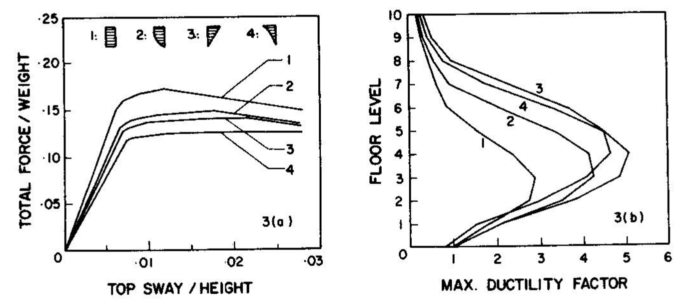

FIG.3 HORIZONTAL FORCE DISTRIBUTION AND STATIC BEHAVIOUR

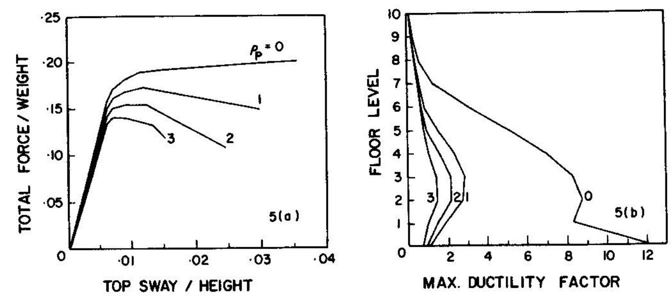

FIG. 5 'P-Δ' FORCES AND STATIC BEHAVIOUR

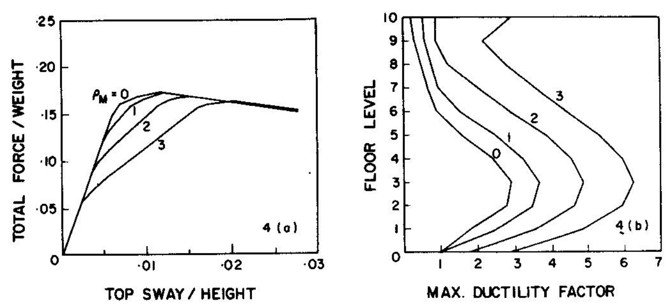

FIG. 4 PRIMARY MOMENTS AND STATIC BEHAVIOUR

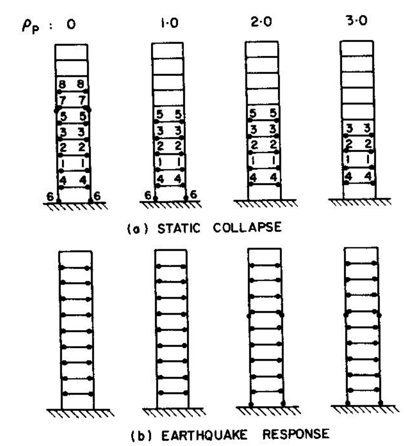

FIG. 6 HINGE FORMATION PATTERNS

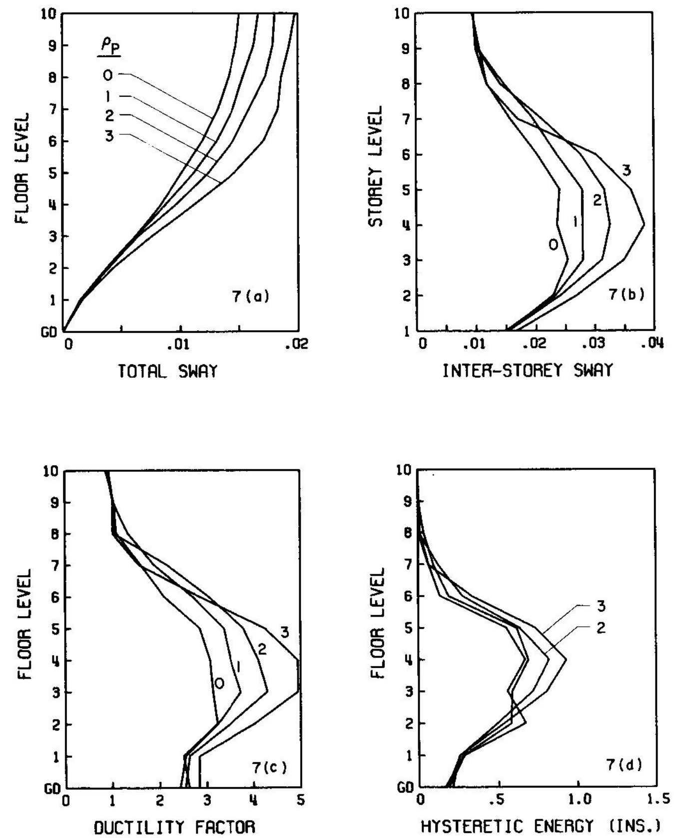

FIG.7 'P - Δ' FORCES AND INELASTIC RESPONSE

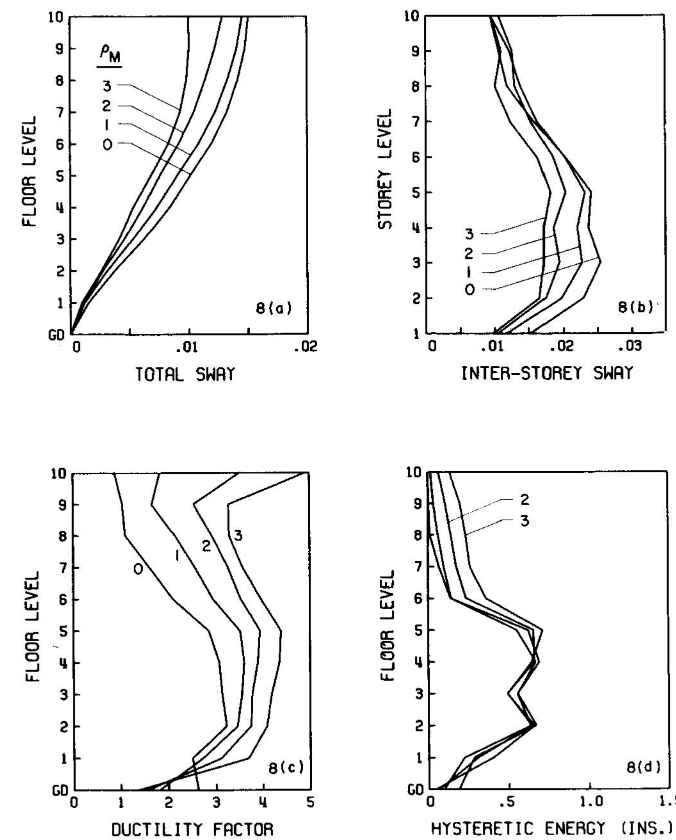

FIG.8 PRIMARY MOMENTS AND INELASTIC RESPONSE

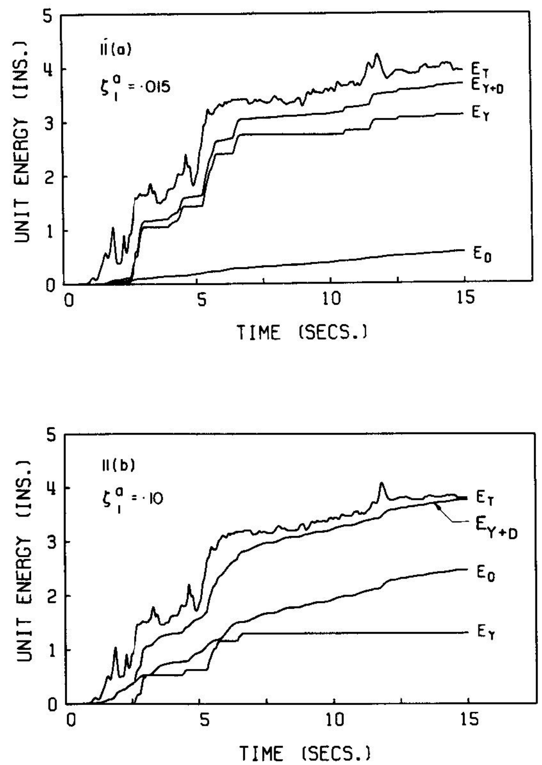

FIG. 11 DAMPING RATIO AND DYNAMIC ENERGY CURVES

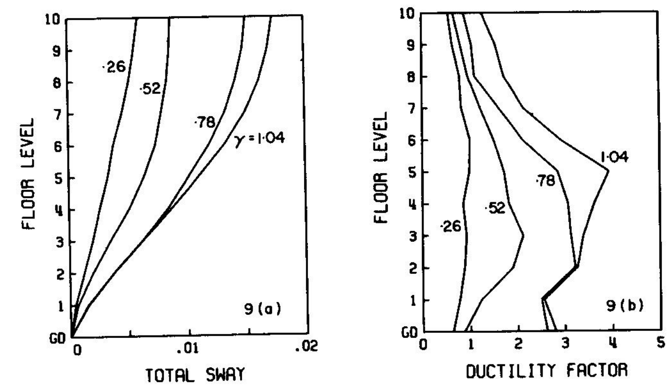

FIG. 9 INTENSITY OF EXCITATION AND INELASTIC RESPONSE

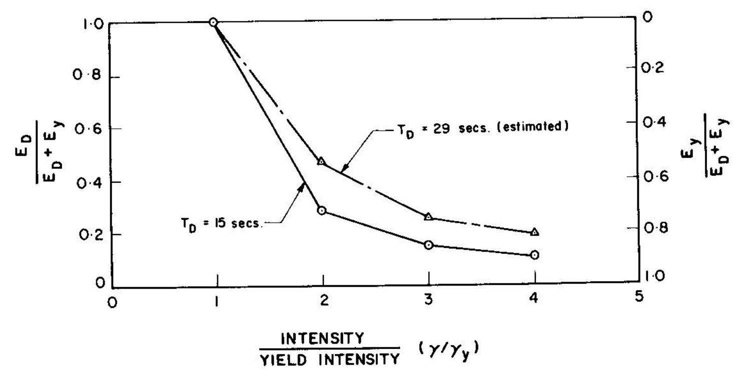

FIG. 10 RELATIVE INTENSITY AND ENERGY DISSIPATION

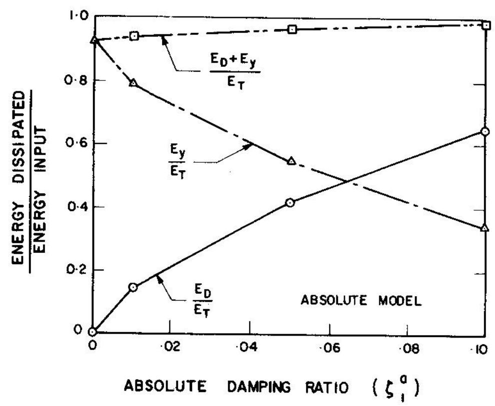

FIG. 12 DAMPING RATIO AND ENERGY DISSIPATION

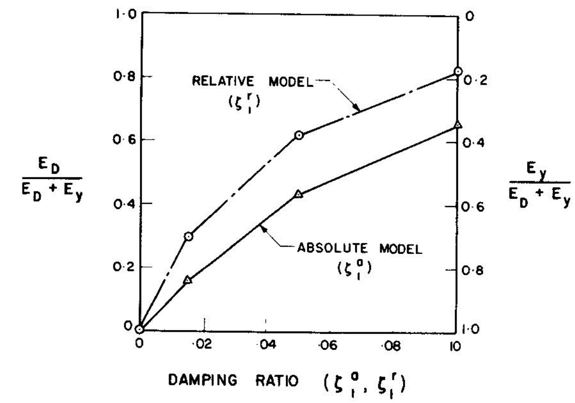

FIG. 13 VISCOUS DAMPING MODEL AND ENERGY DISSIPATION

DISCUSSION OF PAPER NO. 19

INELASTIC BEHAVIOUR OF FRAME STRUCTURES UNDER STATIC AND EARTHQUAKE FORCES

by

O.A. Pekau, R. Green and A.N. Sherbourne

Question by: Y.O. Beredugo

Could the speaker clarify the unit of energy expressed in "inches" used in his paper?

Reply by: O.A. Pekau

I have discussed this point both in the paper itself, as well as in this morning's talk. Energy relationships are expressed in terms of 'unit energies' which give energy per unit total weight of structure. Thus, in lbs/lb reduces to 'inches', for example.